Axion cosmology and domain walls

T. Hiramatsu, M. Kawasaki and KS,
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Abstract

- Reargue axion cosmology which suffers from the domain wall problem
- Two dimensional lattice simulation of domain walls bounded by strings which arise naturally in axion models
- Estimate the decay time of domain walls
- Constrain the model parameters
- Decay of domain wall
  $\rightarrow$ Production of graviational waves

Axions

- Strong CP problem in QCD

$$\mathcal{L}_\theta = \frac{\theta}{32\pi^2} G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$$

  - Violates CP (observation: $\theta \lesssim 10^{-10}$)
  - Why θ is so small?

- Solution: Peccei-Quinn (PQ) mechanism

  Peccei and Quinn (1977)

  - Introduce $U(1)_{PQ}$ symmetry
  - θ is dynamically set into zero

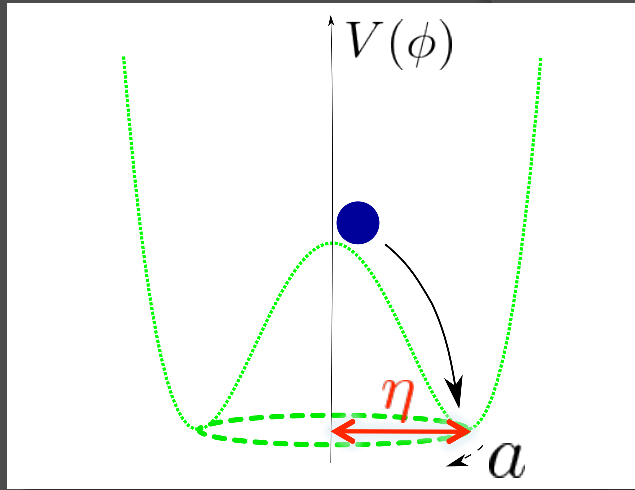
- Pseudo-Nambu-Goldstone boson from spontaneous breaking of $U(1)_{PQ}$

  $\Rightarrow$ Axion

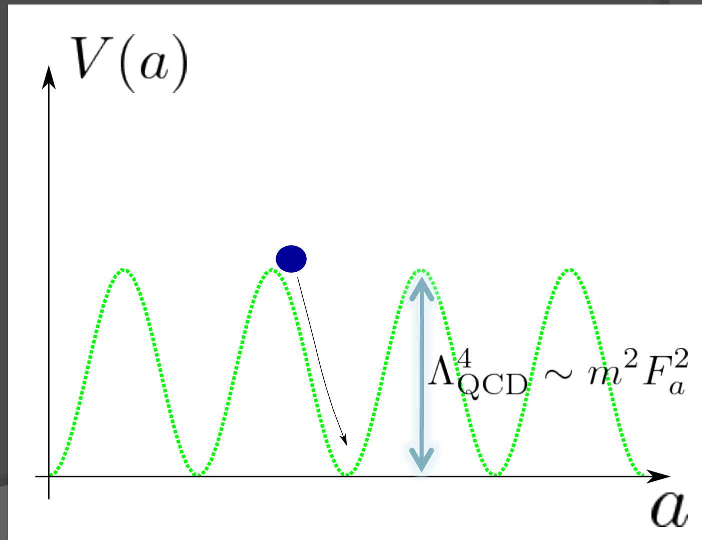
Cosmological Evolution

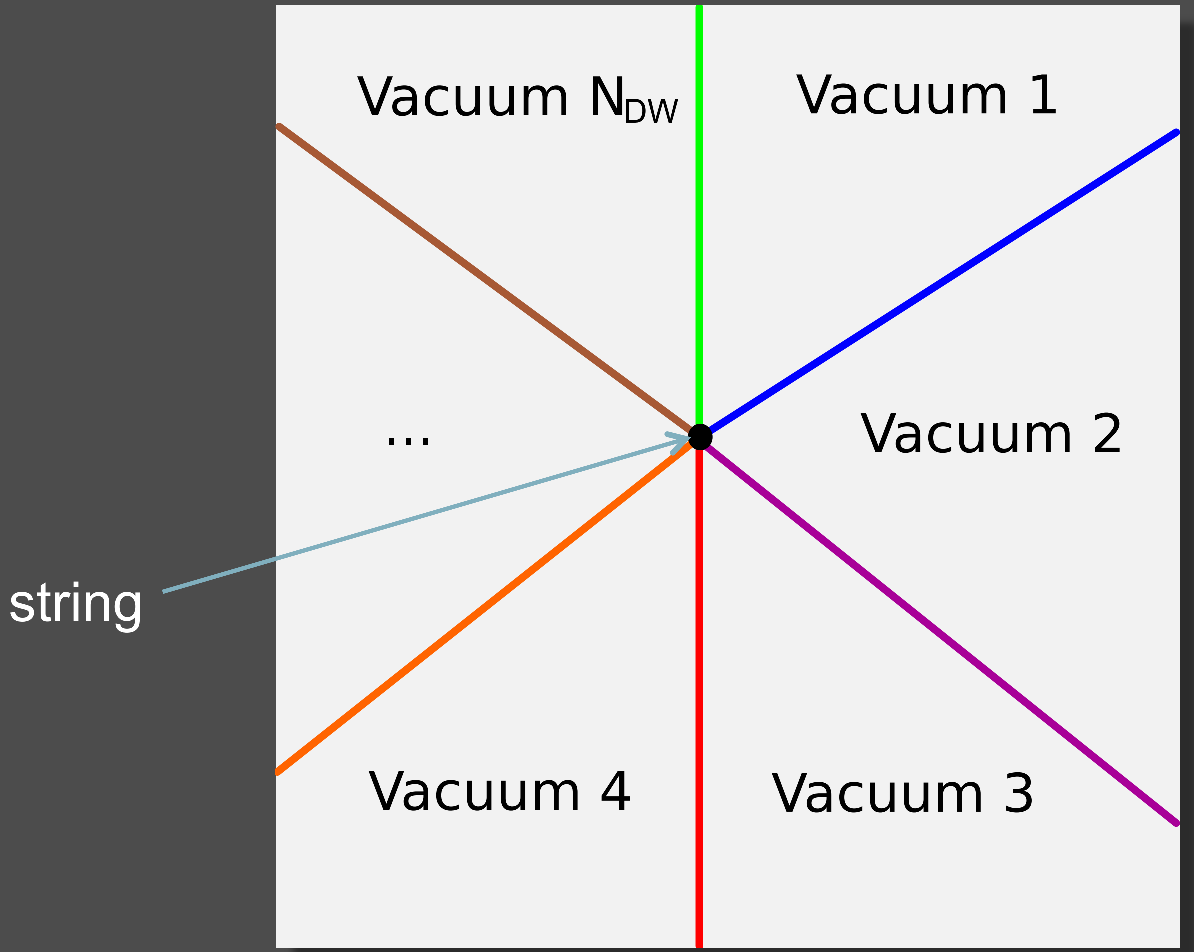
- Peccei-Quinn (PQ) field ϕ
- $T \lesssim F_a$ $\quad F_a = \eta/N_{\rm DW} \sim 10^{10-11}{\rm GeV}$
  - Spontaneous breaking of $U(1)_{PQ}$
  - Formation of cosmic strings

$$\phi = \langle\phi\rangle e^{ia/\eta} = \eta e^{ia/\eta}$$



- $T \sim \Lambda_{\rm QCD}$ $\quad U(1)_{\rm PQ} \to Z_{N_{\rm DW}}$
  - Axion acquires a mass (QCD instanton effect)
- $H \lesssim m_a$ ($T \lesssim 1{\rm GeV}$)
  - Spontaneous breaking of $Z_{N_{DW}}$
  - Formation of domain walls





$$V(a) \sim \Lambda_{\mathrm{QCD}}^4 \left[1 - \cos\left(N_{\mathrm{DW}} a/\eta\right)\right]$$

N_{DW} discrete vacua at

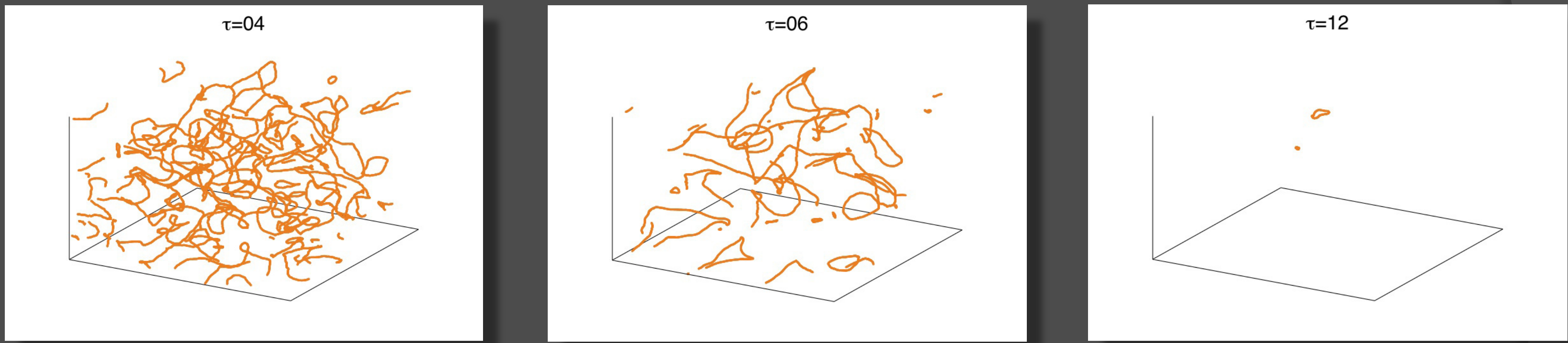
$$\theta = \frac{a}{\eta} = \frac{2\pi k}{N_{\mathrm{DW}}}, \qquad k = 0, 1, \ldots, N_{\mathrm{DW}} - 1$$

Domain Wall Problem

- Domain wall number N_{DW}

$$N_{\rm DW} = {\rm Tr}[Q_{\rm PQ}(q) I(q)]$$: depend on models

- $N_{DW} = 1$; walls quickly disappear



  - Strings decay due to the domain wall tension
  - Axions produced by the decay contribute to the CDM component of the universe [work in progress]

- $N_{DW} > 1$; stable string-wall networks
  - Come to overclose the universe, hence problematic
  - Possible if we introduce a bias Sikivie (1982)

Model

- Potential for the complex scalar

$$V(\phi) = \frac{\lambda}{4}(\phi^*\phi - \eta^2)^2 + \frac{m^2\eta^2}{N_{\rm DW}^2}(1 - \cos N_{\rm DW}\theta) + \delta V$$

cosmic string

domain wall

bias

- The explicit $Z_{N_{DW}}$ breaking term (bias)

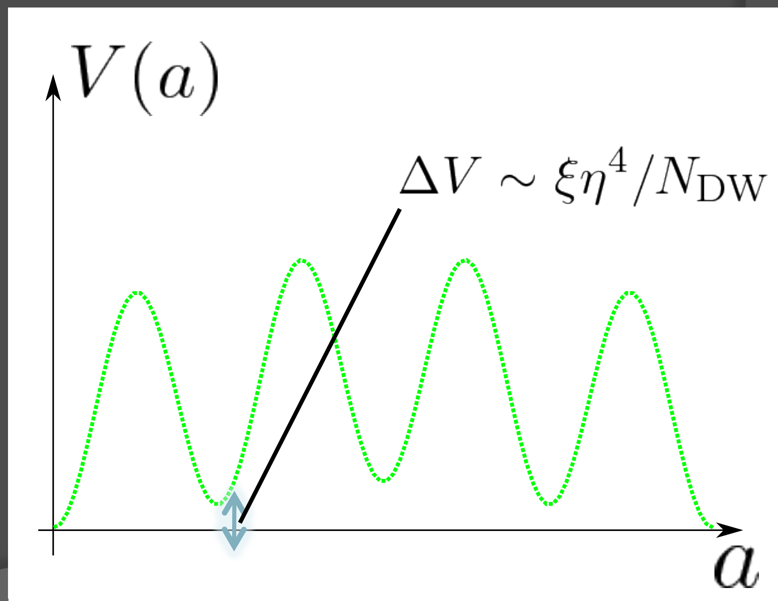
$$\delta V = -\xi\eta^3(\phi^{-i\delta} + \text{h.c.})$$

  - Lifts degenerate vacua

    ⇒ decay of walls

  - ξ : free parameter

    $\xi \ll 1$



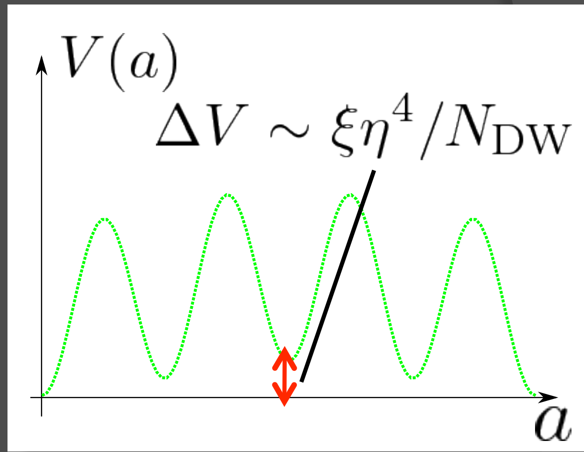
Bias

- $\xi \neq 0$
- Two forces acting on domain walls
  - Tension (straightens the wall)

    $$p_T \sim \sigma/R \sim m\eta^2/N_{\rm DW}^2 R$$

  - Pressure (collapses the wall)

    $$p_V \sim \Delta V \sim \xi\eta^4/N_{\rm DW}$$



- Pressure dominates when

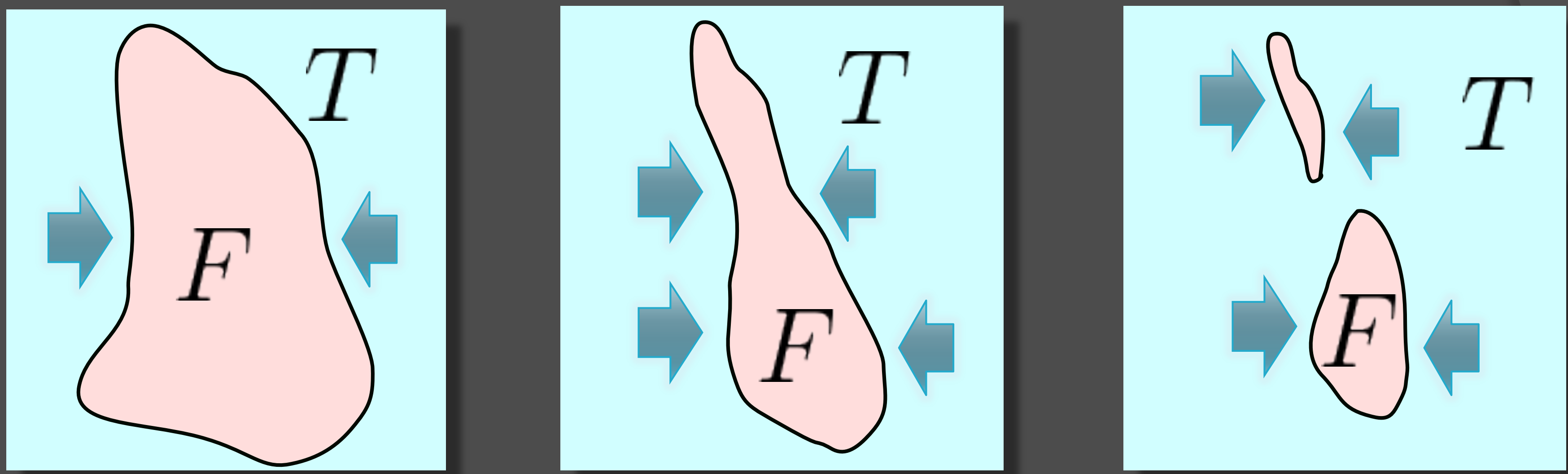
$$p_T \sim p_V \quad \rightarrow \quad R \sim m/N_{\rm DW}\xi\eta^2$$

  - Decay time of walls

    $$t_{\rm dec} \sim R \sim m/N_{\rm DW}\xi\eta^2$$

Collapse of domain walls

$$p_V \sim \Delta V \sim \xi\eta^4/N_{\rm DW}$$



- Due to the volume pressure which caused by energy difference between two vacua (bias) .

Setup of Lattice Simulations

- Solve the classical field equations for ϕ on 2D lattice

$$\bar{\phi}_1'' - \nabla^2\bar{\phi}_1 = -\lambda\bar{\phi}_1(|\bar{\phi}|^2 - a^2) + 2a^3\xi\cos\delta + \frac{a^4 m^2}{N_{\rm DW}|\bar{\phi}|}\sin\theta\sin N_{\rm DW}\theta \qquad \phi = \phi_1 + i\phi_2$$

$$\bar{\phi}_2'' - \nabla^2\bar{\phi}_2 = -\lambda\bar{\phi}_2(|\bar{\phi}|^2 - a^2) + 2a^3\xi\cos\delta - \frac{a^4 m^2}{N_{\rm DW}|\bar{\phi}|}\cos\theta\sin N_{\rm DW}\theta \qquad \phi \equiv \bar{\phi}/a$$

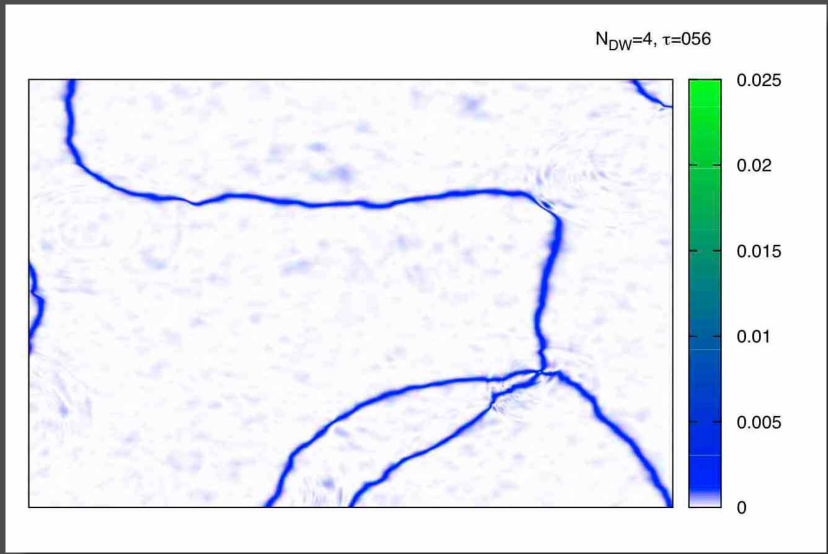
- Input parameters

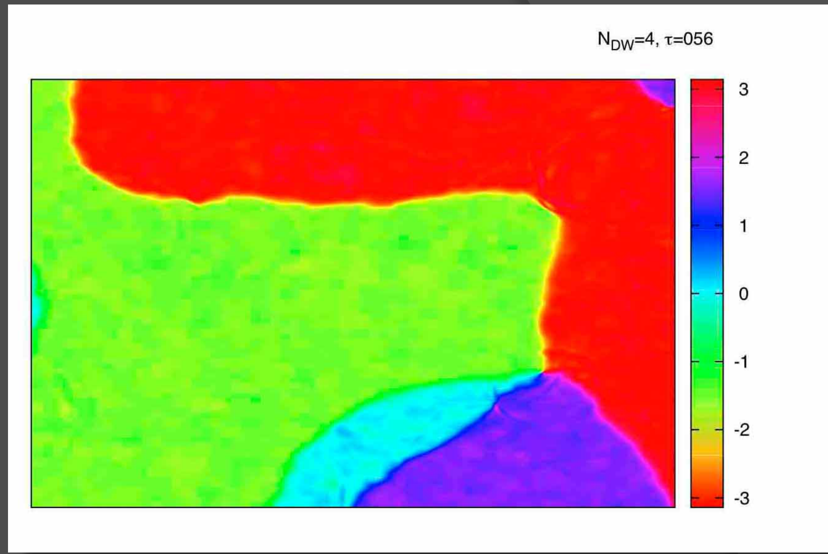
$$d\tau = dt/a \quad \phi' = \frac{d\phi}{d\tau}$$

(τ : conformal time)

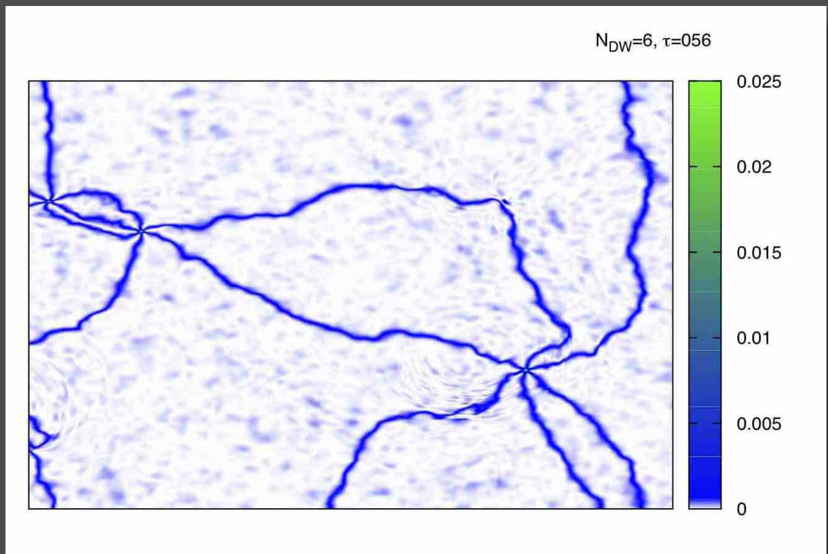
Scheme	4th Runge-Kutta
Number of grid	4096×4096
Era	Radiation dominated
Initial time	2 (in unit of η^{-1})
Final time	110
Time resolution	0.01
Box size	230
λ	0.1
m/η	0.1
$N_{\rm DW}$, ξ	varying

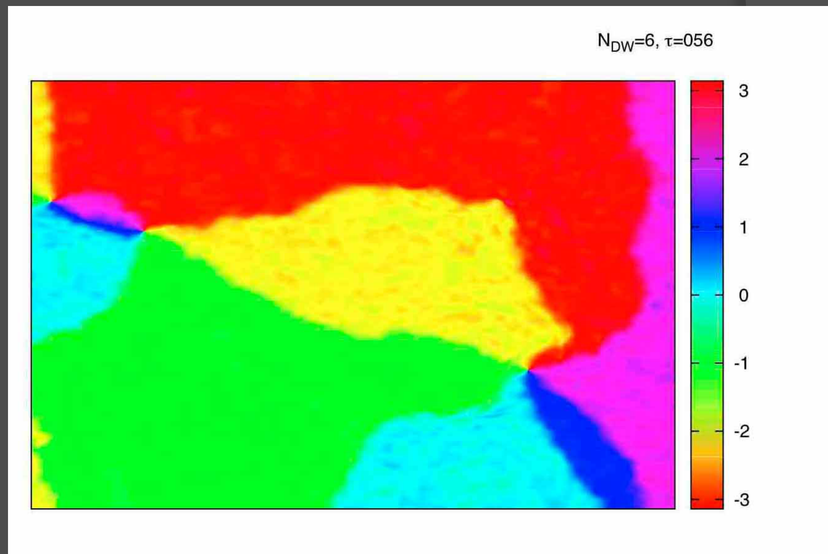
N_{DW}=4





N_{DW}=6

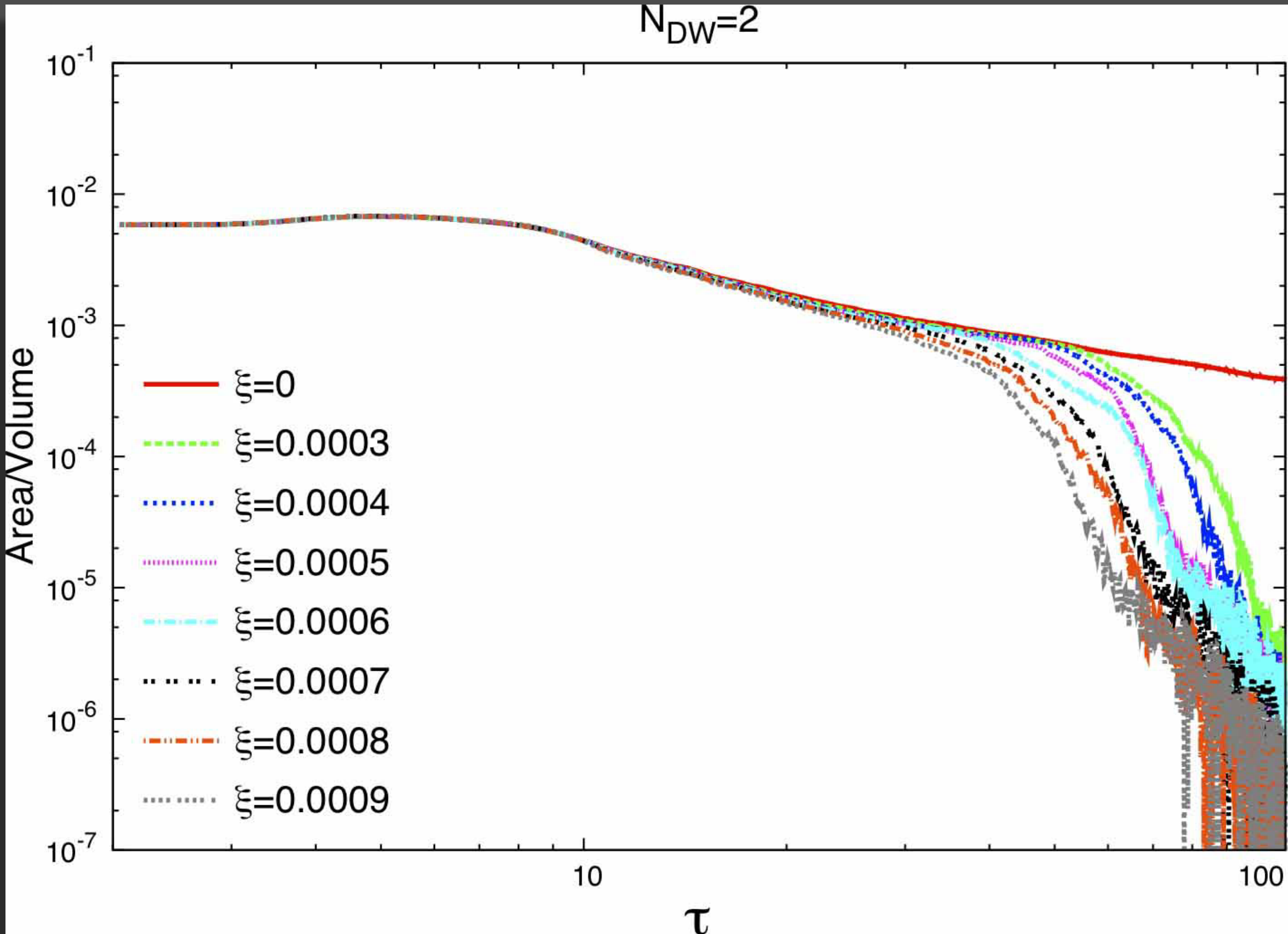




Potential energy

Phase θ

Time evolution of the area density



Fitted as $$t_{\rm dec} \simeq 18 \times \left(\frac{m}{N_{\rm DW}\xi\eta^2} \right)$$

Bounds for ξ

Neutron Electric Dipole Moment (NEDM)

- The bias term δV spoils the PQ solution to the strong CP problem
- Shifts the θ value from zero

$$\theta \sim \frac{\xi \eta^2}{m^2} < 10^{-10}$$

- Observation of NEDM $\Rightarrow$ Upper bound for ξ

Overclosure bound

- Wall dominates when

$$\rho_{\rm wall} \simeq \sigma/t \sim \rho_c \simeq 1/Gt^2$$

$$\Rightarrow \quad t_{\rm WD} \sim 1/G\sigma$$

- Require $t_{\rm dec} < t_{\rm WD}$ $\Rightarrow$ Lower bound for ξ

simulation

Cold axions from domain walls

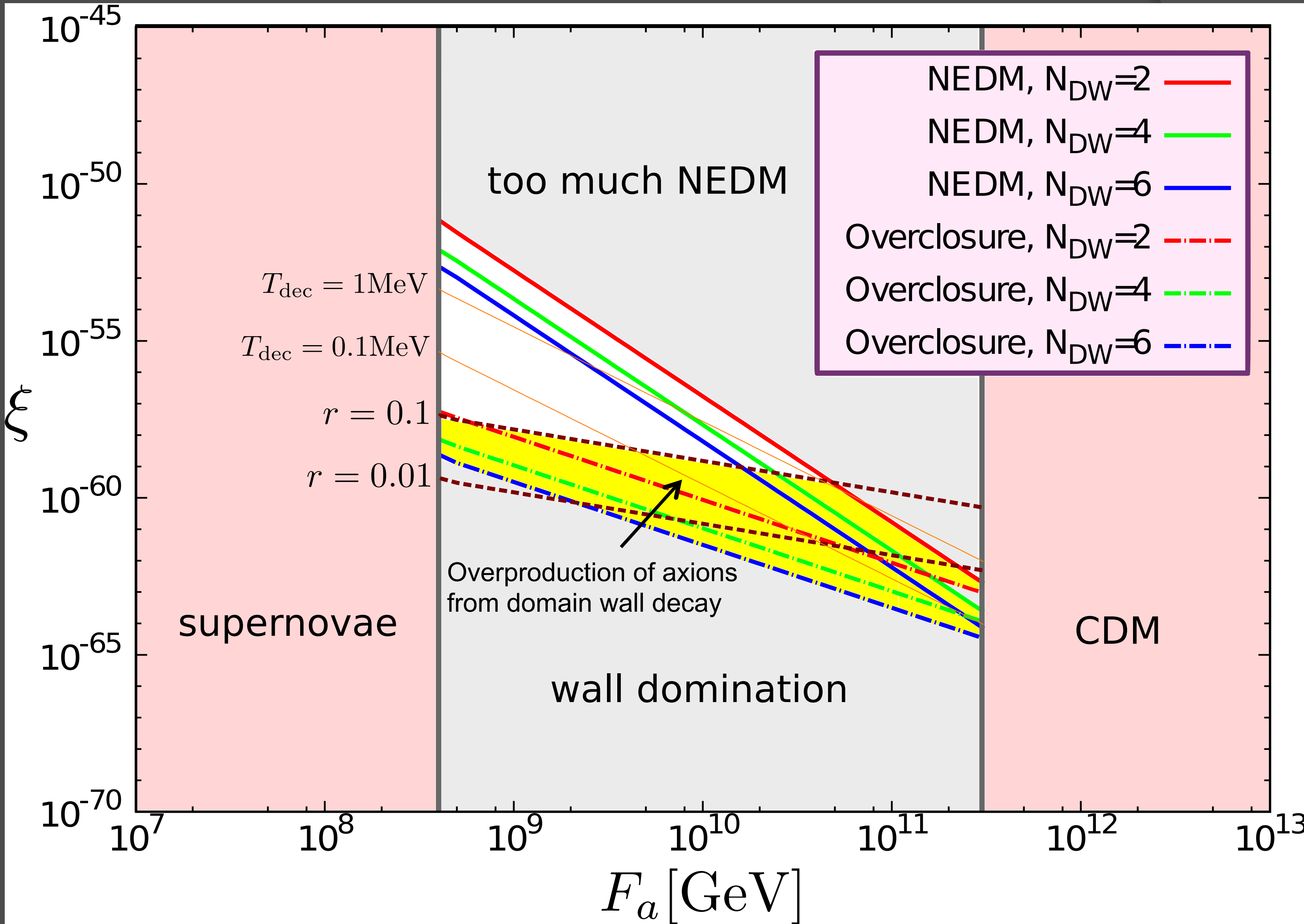
- Decay of domain walls $\Longrightarrow$ production of axions
  - The fraction r of the wall energy goes into axion radiations $\rho_a(t_{\rm dec}) = r\rho_{\rm wall}(t_{\rm dec})$
  - Radiated axions are barely relativistic with Lorentz factor $\gamma \simeq 60$ Nagasawa and Kawasaki (1994), Chang, Hagmann and Sikivie (1999)

    $\Longrightarrow$ become CDM component of the universe

- Abundance of cold axions from domain walls at the time of equality between matter and radiation

$$\Omega_a(t_{\rm eq}) \equiv \frac{\rho_a(t_{\rm dec})}{\rho_c(t_{\rm eq})} \approx 3\times 10^{-29} \times r\xi^{-1/2} N_{\rm DW}^{-3/2} \left(\frac{60}{\gamma}\right)\left(\frac{0.15}{\Omega_M h^2}\right)\left(\frac{10^{12}{\rm GeV}}{F_a}\right)^{1/2} < \frac{1}{2}$$

  - Another lower bound on ξ
  - r must be small



Constraints become much severe if r is not suppressed

Gravitational Waves from DW

- Interactions of domain wall networks
  - Produce gravitational waves (GWs)
    - Become stochastic gravitational wave background

- Intensity of GWs depends on
  - Mass energy of the wall $G\sigma^2 \sim F_a^4 (m/M_P)^2$
  - Life time of the wall $t_{\rm dec} \sim m/\xi F_a^2$

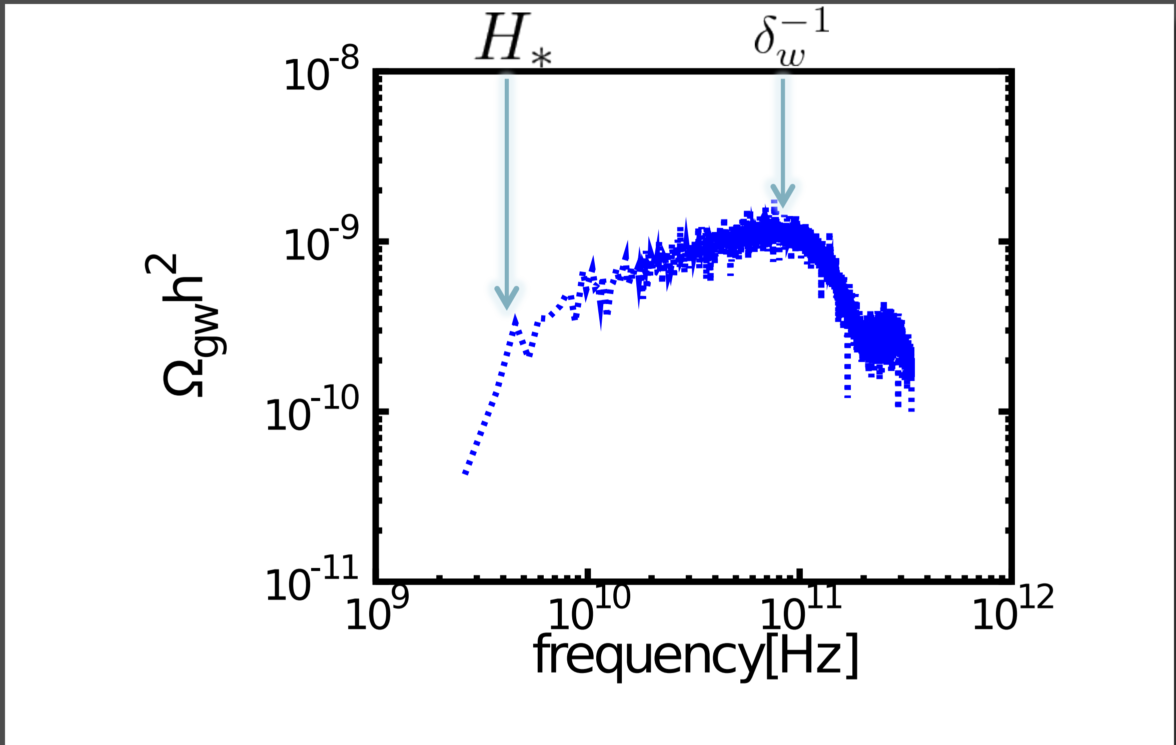
Hiramatsu, Kawasaki and KS (2010)

- Small ξ
  - long life time
  - likely to produce GWs with large amplitude

Spectrum of GWs

- Z_2 model of real scalar field

Hiramatsu, Kawasaki and KS, arXiv:1002.1555
Kawasaki and KS, arXiv:1102.5628



- Slope changes at the frequency corresponding to
  - Wall width : $f_* \sim \delta_w^{-1}$
  - Hubble scale at $t_{\rm dec}$: $f_* \sim H_* \sim t_{\rm dec}^{-1}$
- Nearly flat spectrum in the intermediate scales

Observable?

- Emission of GWs is terminated at $t_* \simeq t_{\rm dec}$

Intensity

$$\Omega_{\rm gw}h^2 \equiv \frac{1}{\rho_c(t_0)}\frac{d\rho_{\rm gw}(t_0)}{d\log f} \sim 10^{-5}\frac{\rho_{\rm gw}(t_*)}{\rho_c(t_*)}$$

$$\rho_{\rm gw}(t_*) \sim G\sigma^2$$
$$\rho_c(t_*) \sim 1/Gt_*^2$$

$$\sim 5\times 10^{-12}\times\left(\frac{4}{N_{\rm DW}}\right)^6\left(\frac{10^{-58}}{\xi}\right)^2\left(\frac{10^{10}{\rm GeV}}{F_a}\right)^4$$

Spectrum extends from

$$f = \frac{a(t_*)}{a(t_0)}H_* \sim 2\times 10^{-11}\times\left(\frac{N_{\rm DW}}{4}\right)^{3/2}\left(\frac{\xi}{10^{-58}}\right)^{1/2}\left(\frac{F_a}{10^{10}{\rm GeV}}\right)^{3/2}{\rm Hz}$$

to

$$f = \frac{a(t_*)}{a(t_0)}m \sim 6\times 10^2\times\left(\frac{4}{N_{\rm DW}}\right)^{3/2}\left(\frac{10^{-58}}{\xi}\right)^{1/2}\left(\frac{10^{10}{\rm GeV}}{F_a}\right)^{5/2}{\rm Hz}$$

Cf.
DECIGO $\Omega_{\rm gw}h^2 \sim 10^{-20}$ at $f \sim 10^{-1}{\rm Hz}$

- Future experiments can detect signals → probe axion models

Conclusion

- Domain wall problem can be avoided if domain walls decay before they dominate the energy density of the universe
- 2 dim Lattice simulation
  - Confirm the decay of the network
  - Estimate the time when walls decay
- Observational constraints are severe but do not completely rule out the scenario
  - Signals in future GW experiments can be used to probe the models with $N_{DW}>1$ (or exclude them)

Future prospects

- We need a detailed investigation about relic radiations produced by axionic domain walls
- $N_{DW}=1$ scenario
  - Estimation of axion CDM abundance produced by domain walls
- $N_{DW}>1$ scenario
  - Estimation of axion CDM and GW abundance produced by domain walls
  - Calculation of the GW spectrum
  - Determine uncertain factor $\mathcal{r}$
- Develop analysis in full 3D simulation [work in progress]

Appendix

Initial Conditions

- Treat ϕ_1 and ϕ_2 as two independent real scalar fields with correlation function $\phi = \phi_1 + i\phi_2$

$$\langle \phi_i(\mathbf{k})\phi_i(\mathbf{k}')\rangle = \frac{1}{2k}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

$$\langle \dot{\phi}_i(\mathbf{k})\dot{\phi}_i(\mathbf{k}')\rangle = \frac{k}{2}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}') \qquad (i=1,2)$$

- No correlation in the k space

→Generate $\phi_i(\mathbf{k})$ as Gaussian with

$$\langle|\dot{\phi}(\mathbf{k})|^2\rangle = \frac{k}{2}V_b \qquad \langle|\phi(\mathbf{k})|^2\rangle = \frac{1}{2k}V_b$$

$$\langle\phi(\mathbf{k})\rangle = \langle\dot{\phi}(\mathbf{k})\rangle = 0 \qquad V_b \simeq (2\pi)^3\delta^{(3)}(0)$$

: volume of the simulation box

→ Fourier transform to obtain $\phi_i(\mathbf{x})$ and $\dot{\phi}_i(\mathbf{x})$

Comments on the numerical study

- One must consider three extremely different length scales
  - Core of the sting
    $\delta_s \sim 1/\sqrt{\lambda}\eta \sim \text{const.} > \text{lattice spacing} \sim a(t)$
  - Width of the wall
    $\delta_w \sim m^{-1} \sim \text{const.} > \text{lattice spacing} \sim a(t)$
  - Hubble radius
    $H^{-1} \sim t < \text{simulation box}$
- At the final time of the simulation, the core of the string is marginally resolvable

$$\frac{H^{-1}}{a(t)\delta x} = \frac{2N}{b} t^{1/2} \simeq 1024, \quad \frac{\delta_w}{a(t)\delta x} = \frac{N}{bm} t^{-1/2} \simeq 3.2, \quad \frac{\delta_s}{a(t)\delta x} = \frac{N}{b\lambda^{1/2}} t^{-1/2} \simeq 1.01$$

at $t = t_f = 1601\eta^{-1}$

$\delta x = b/N$: lattice spacing

Effect on Big Bang Nucleosynthesis (BBN)

- Domain walls dominates the energy density of the universe at the temperature

$$T \simeq 8 \times 10^{-2} \times \left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)^{1/2} \mathrm{MeV}$$

→ The wall domination occurs after the BBN epoch

- During the BBN epoch Domain walls contributes as an extra particle d.o.f.

$$\rho_{\mathrm{extra}}(t_{\mathrm{BBN}}) = \frac{\pi^2}{30}\frac{7}{8}(N_\nu - 3)T_{\mathrm{BBN}}^4 = \rho_{\mathrm{wall}}(t_{\mathrm{BBN}}) = \sigma H_{\mathrm{BBN}}$$

- Observations indicate $N_\nu \lesssim 4$
- However, the contribution from domain walls is negligible

$$N_\nu - 3 = 8.4 \times 10^{-2} \times \left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)$$

Scaling Solution

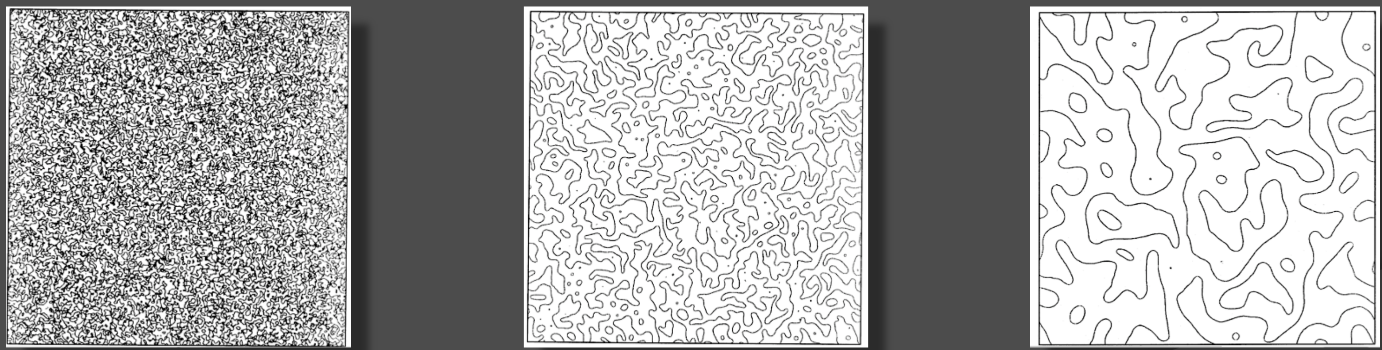
Press, Ryden, and Spergel (1989)

- $\xi = 0$
- One wall per one Hubble radius
  - $L \sim R \sim H^{-1} \sim t$
    where L is the distance of two neighboring walls
    and R is the curvature radius of walls
  - Energy density

$$\rho_w \sim \sigma R^2 / R^3 \sim \sigma / t$$

Surface mass density of the wall

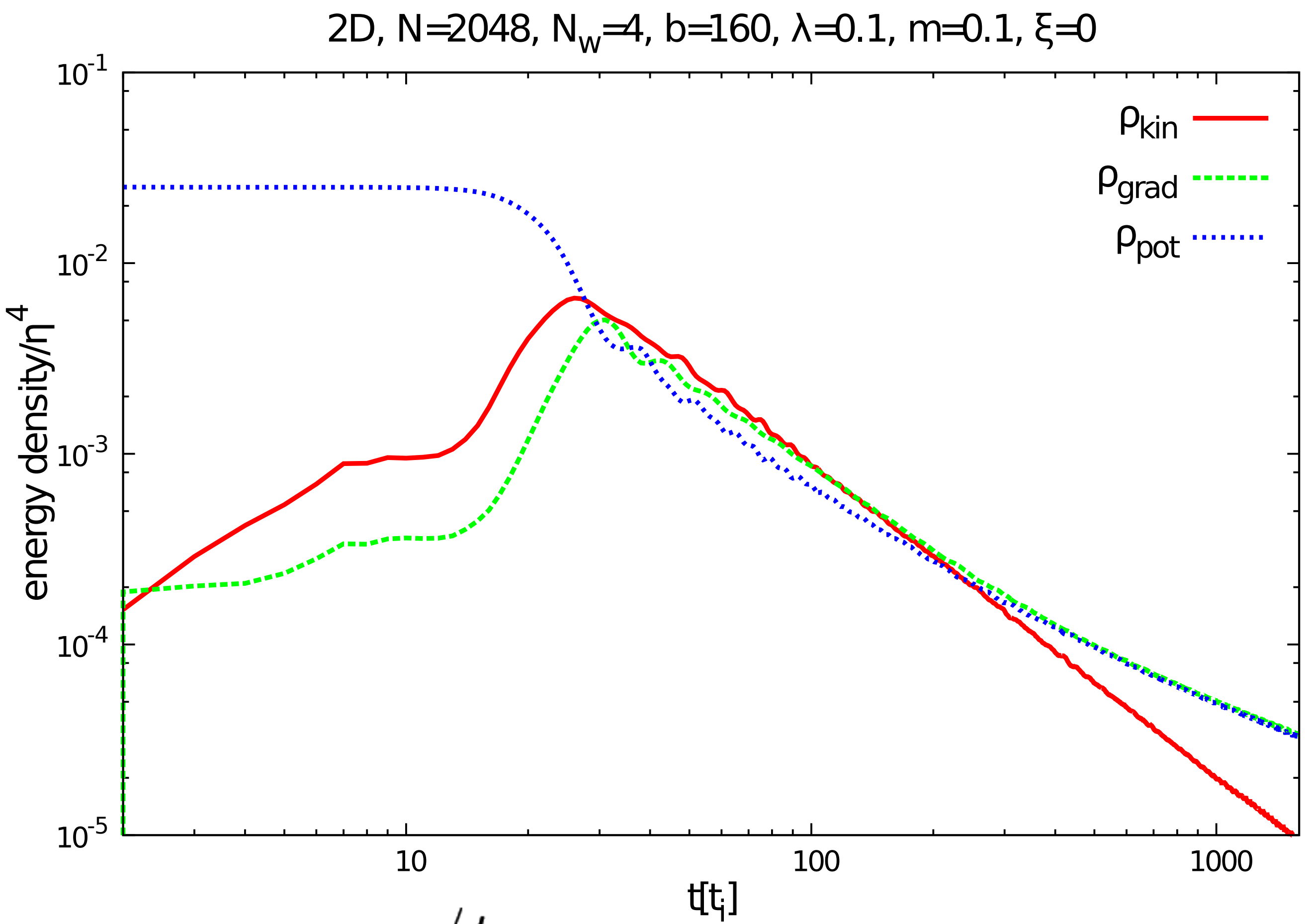
$$\sigma \simeq 9 m F_a^2$$



Hubble radius / Box size

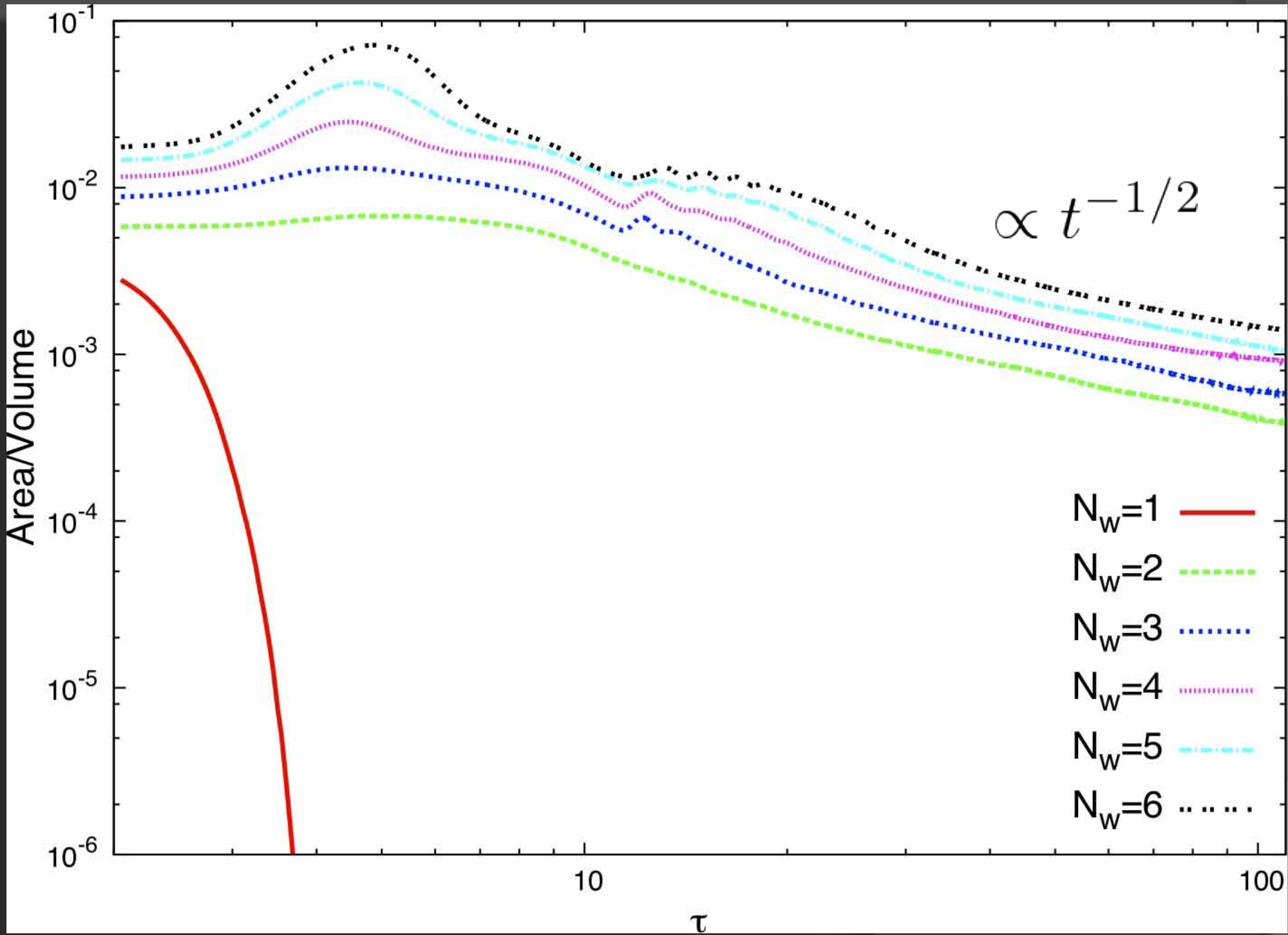
1/100 1/30 1/10

Energy density

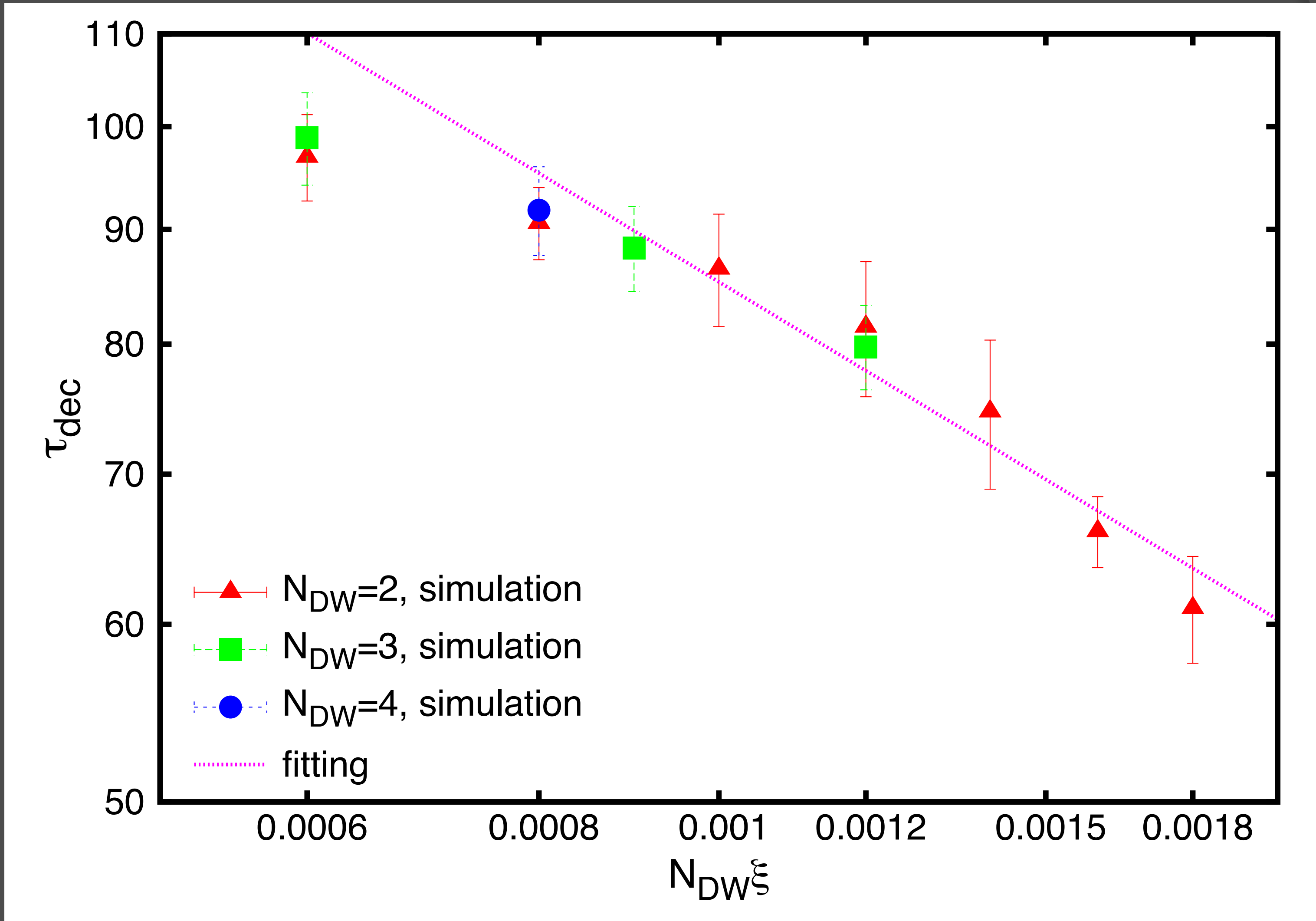


$$\rho_{\rm grad} \sim \rho_{\rm pot} \sim \sigma/t$$

Area Density ($\xi = 0$) $\quad A/V \propto 1/t^{1/2}$



Relation between $\tau_{\rm dec}$ and $N_{\rm DW}\xi$

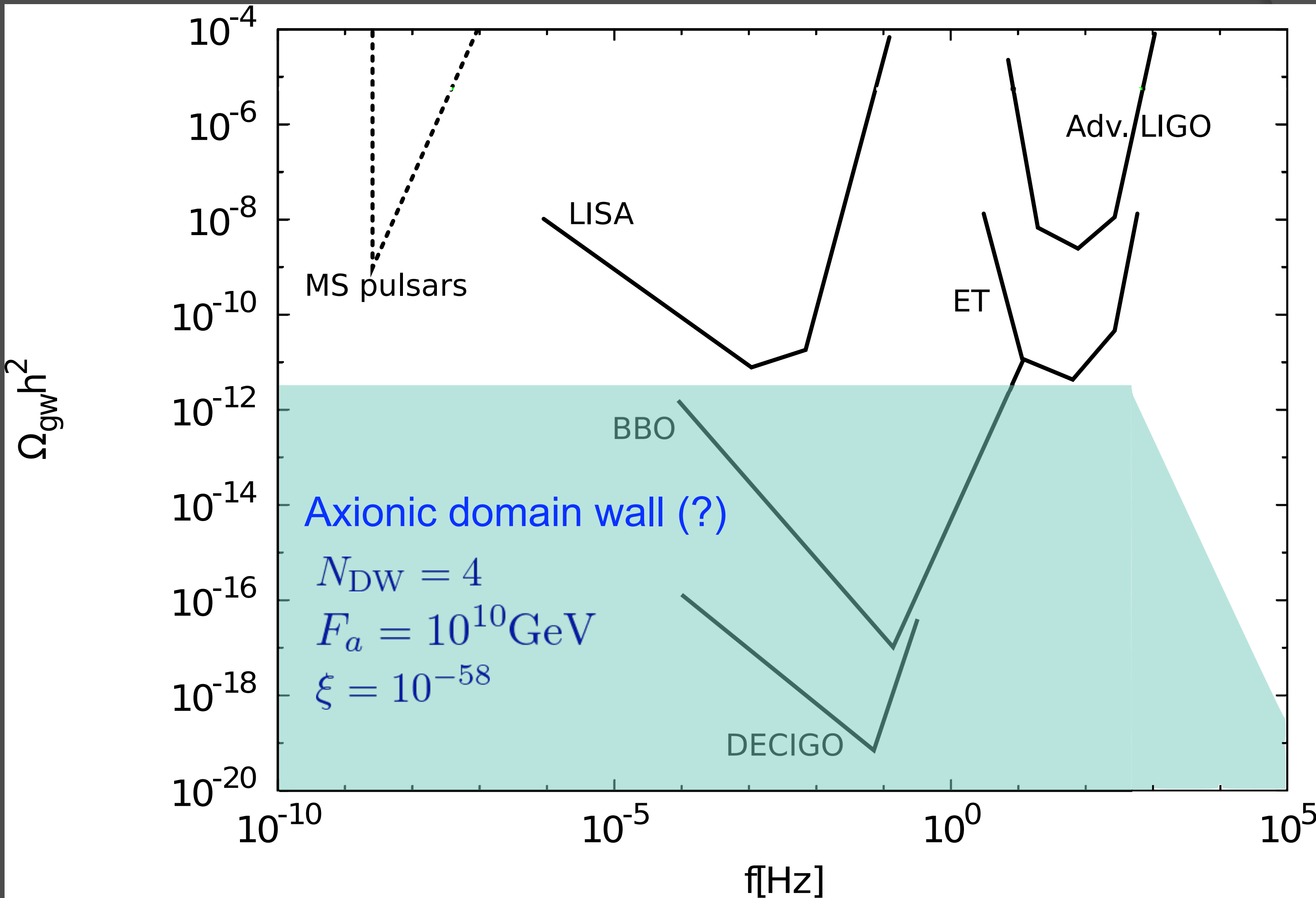


Solutions to the DW Problem

- $N_{DW} > 1$ ⟹ Domain walls are problematic
- Possible solutions:
  - Embed $Z_{N_{DW}}$ in the center of another continuous group

    Lazarides and Shafi (1982)
    - Can be realized only for a particular set of charge assignments
  - Inflation after PQ phase transition
    - Constraints from isocurvature perturbations
  - Unstable domain walls Sikivie (1982)
    - Domain walls decay before overclose the energy density of the universe
      - Possibility? Constraints? Any implications for observations?

Schematics



10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
$\Omega_{gw}h^2$
10^{-10}
10^{-5}
10^{0}
10^{5}
f[Hz]
MS pulsars
LISA
Adv. LIGO
ET
BBO
DECIGO
Axionic domain wall (?)
$N_{\rm DW} = 4$
$F_a = 10^{10}$GeV
$\xi = 10^{-58}$